Econ 802

First Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Let $y_1 \leq 0$ be an input and let $y_2 \geq 0$ be an output. The production plan $y = (y_1, y_2)$ is feasible if and only if $0 \leq y_2 \leq 4 - (y_1 + 2)^2$.

(a) Draw a graph of the production possibilities set Y. Is it closed? bounded? strictly convex? Briefly justify your answers.

(b) Change notation so the input is the scalar $x \geq 0$ and the output is the scalar $y \geq 0$. The technology is the same. Describe the input requirement sets V(0), V(2), and V(4) mathematically. Are these sets monotonic? Briefly justify your answers.

(c) Assume the prices of both goods are positive. Does the firm's profit maximization problem always, sometimes, or never have a solution? Does the cost minimization problem always, sometimes, or never have a solution? Justify your answers.

2. The following questions all deal with <u>profit maximization</u>.

(a) Consider the production function $y = \min \{ax_1, bx_2\}$ with $a > 0$, $b > 0$, and $x \geq 0$. What restrictions must be imposed on the prices $(p, w_1, w_2) > 0$ to ensure that the profit maximization problem has a solution? Under what conditions would the solution be unique? Explain.

(b) Consider the production function $y = ax_1 + bx_2$ with $a > 0$, $b > 0$, and $x \geq 0$. What restrictions must be imposed on the prices $(p, w_1, w_2) > 0$ to ensure that the profit maximization problem has a solution? Under what conditions would the solution be unique? Explain.

(c) Consider the production function $y = x_1^a x_2^b$ with $a > 0$, $b > 0$, and $x \geq 0$. There are <u>no</u> restrictions on the prices $(p, w_1, w_2) > 0$. Under what conditions does the profit maximization problem <u>definitely have</u> a solution? Under what conditions does it <u>definitely not have</u> a solution? Use some math to justify your answers. Hint: use what you know about the general form of the cost function c(w, y).

3. Use the notation $y = (y_1 \, . \, . \, y_n)$ for production plans and $p = (p_1 \, . \, . \, p_n)$ for prices, where inputs are negative numbers, outputs are positive numbers, and all prices are strictly positive.

(a) Prove that the profit function $\pi(p)$ is convex.

(b) State Hotelling's Lemma and prove that it is true.

(c) Consider the case $n = 2$, where there is one input and one output. Suppose you observe the firm's production plan and price vector in the time periods $t = 1, 2$. Construct a numerical example that would violate the Weak Axiom of Profit Maximization, and explain your reasoning using a graph.

4. The following questions all deal with cost minimization.

(a) Consider four assumptions: (i) the necessary second order condition holds at all points; (ii) the production function is quasi-concave; (iii) the production function is strictly quasi-concave; (iv) the sufficient second order condition holds at all points. What do we gain by making assumption (ii) instead of (i); assumption (iii) instead of (ii); and assumption (iv) instead of (iii)? Explain briefly in each case.

(b) When working with a cost minimization problem, economists sometimes invert a bordered Hessian matrix. Explain what problem an economist would be trying to solve by doing this; what the mathematical steps are that lead up to the inversion of the matrix; and what the conclusions are. You do not need to invert a matrix.

(c) Consider a short run average cost function $SAC(y, x_f)$ where y is output and x_f is the level of an input that is fixed in the short run but variable in the long run. Use a graph to explain why it would not generally make economic sense for the long run average cost curve to pass through the minimum point of the SAC curve, and support your answer using mathematics. Can there be any exception to this rule?

5. Here are some miscellaneous questions.

(a) Define the elasticity of substitution. Then imagine that an undergraduate student asks you why anyone cares about this concept. What would you say? Use some math to support your answer.

(b) Prove that no homogeneous production function of any degree gives a U-shaped long run average cost curve. Explain your reasoning.

(c) A firm maximizes $pf(x) - wx$ where $p > 0$ is output price, $f(x)$ is the production function, $x = (x_1 \, . \, . \, x_n) \geq 0$ is a vector of inputs, and $w = (w_1 \, . \, . \, w_n) > 0$ is a vector of input prices. Assume the output supply function $y(p, w)$ is well-defined. Use any reasonable method and any reasonable assumptions to show that $y(p, w)$ is a non-decreasing function of the output price p when w is held constant.